2.1 Use Inductive Reasoning

Goal • Describe patterns and use inductive reasoning.

Your Notes

VOCABULARY

Conjecture A conjecture is an unproven statement that is based on observations.

Inductive Reasoning Inductive reasoning is the process of finding a pattern for specific cases and then writing a conjecture for the general case.

Counterexample A counterexample is a specific case for which the conjecture is false.

Example 1 *Describe a visual pattern*

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.
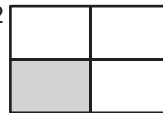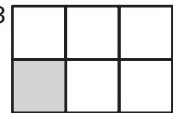
Figure 1   Figure 2   Figure 3

Solution

Each rectangle is divided into twice as many equal regions as the figure number. Sketch the fourth figure by dividing the rectangle into eighths . Shade the section just below the horizontal segment at the left .
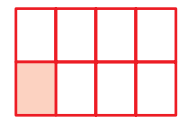
Figure 4

✔ *Checkpoint* **Complete the following exercise.**

1. Sketch the fifth figure in the pattern in Example 1.

Figure 5

> Three dots (. . .) tell you that the pattern continues.

Example 2 *Describe the number pattern*

Describe the pattern in the numbers −1, −4, −16, −64, ... Write the next three numbers in the pattern.

Notice that each number in the pattern is __four__ times the previous number.

−1,   −4,   −16,   −64, . . .

× _4_   × _4_   × _4_   × _4_

The next three numbers are __−256, −1024, and −4096__.

Example 3 *Make a conjecture*

Given five noncollinear points, make a conjecture about the number of ways to connect different pairs of the points.

Make a table and look for a pattern. Notice the pattern in how the number of connections __increases__. You can use the pattern to make a conjecture.

Number of points	1	2	3	4	5
Picture	•	•—•	△	⊠	✪
Number of connections	0	1	3	6	?

+ _1_   + _2_   + _3_   + _?_

Conjecture You can connect five noncollinear points __6 + 4__, or __10__ different ways.

✔ *Checkpoint* **Complete the following exercises.**

2. Describe the pattern in the numbers 1, 2.5, 4, 5.5, . . . and write the next three numbers in the pattern.

The numbers are increasing by 1.5; 7, 8.5, 10.

3. Rework Example 3 if you are given six noncollinear points.

15 different ways

Example 4 *Make and test a conjecture*

Numbers such as 1, 3, and 5 are called consecutive odd numbers. Make and test a conjecture about the sum of any three consecutive odd numbers.

Step 1 Find a pattern using groups of small numbers.

$$1 + 3 + 5 = \underline{\ 9\ }$$
$$= 3 \cdot 3$$
$$5 + 7 + 9 = \underline{\ 21\ }$$
$$= \underline{\ 7\ } \cdot 3$$

$$3 + 5 + 7 = \underline{\ 15\ }$$
$$= \underline{\ 5\ } \cdot 3$$
$$7 + 9 + 11 = \underline{\ 27\ }$$
$$= \underline{\ 9\ } \cdot 3$$

Conjecture The sum of any three consecutive odd numbers is three times the second number .

Step 2 Test your conjecture using other numbers.

$$-1 + 1 + 3 = \underline{\ 3\ } = \underline{\ 1\ } \cdot 3 \checkmark$$
$$103 + 105 + 107 = \underline{\ 315\ } = \underline{\ 105\ } \cdot 3 \checkmark$$

✔ *Checkpoint* Complete the following exercise.

4. Make and test a conjecture about the sign of the product of any four negative numbers.

   The result of the product of four negative numbers is a positive number;
   $(-1)(-2)(-5)(-1) = 10.$

Example 5 *Find a counterexample*

A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The difference of any two numbers is always smaller than the larger number.

To find a counterexample, you need to find a difference that is greater than the larger number.
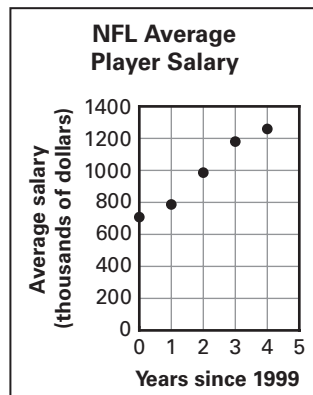
$$8 - (-4) = \underline{\ 12\ }$$

Because $12 \not< 8$, a counterexample exists. The conjecture is false.

Example 6   *Making conjectures from data displays*

The scatter plot shows the average salary of players in the National Football League (NFL) since 1999. Make a conjecture based on the graph.

NFL Average Player Salary

Solution

The scatter plot shows that the values __increased__ each year. So, one possible conjecture is that the average player in the NFL is earning __more__ money today than in 1999.

✔ *Checkpoint* **Complete the following exercises.**

5. Find a counterexample to show that the following conjecture is false.

Conjecture The quotient of two numbers is always smaller than the dividend.

$$\dfrac{4}{\frac{1}{2}} = 8$$

6. Use the graph in Example 6 to make a conjecture that *could* be true. Give an explanation that supports your reasoning.

The average salary of an NFL player in future years will be higher than the previous year; the average salary of an NFL player increased for the 5 years from 1999 to 2003.

Homework