2.2 Analyze Conditional Statements

Goal • Write definitions as conditional statements.

Your Notes

VOCABULARY

Conditional statement A logical statement that has two parts, a hypothesis and a conclusion.

If-then form A form of a conditional statement in which the "if" part contains the hypothesis and the "then" part contains the conclusion.

Hypothesis A hypothesis is the "if" part of a conditional statement.

Conclusion A conclusion is the "then" part of a conditional statement.

Negation The negation of a statement is the opposite of the original statement.

Converse The converse of a conditional statement is formed by switching the hypothesis and conclusion.

Inverse The inverse of a conditional statement is formed by negating both the hypothesis and conclusion.

Contrapositive The contrapositive of a conditional statement is formed by writing the converse and then negating both the hypothesis and conclusion.

Equivalent statements Equivalent statements are two statements that are both true or both false.

Perpendicular lines Two lines that intersect to form a right angle are perpendicular lines.

Biconditional statement A statement that contains the phrase "if and only if."

Your Notes

Example 1 *Rewrite a statement in if-then form*

Rewrite the conditional statement in if-then form.

All vertebrates have a backbone.

Solution

First, identify the hypothesis and the conclusion. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

All vertebrates have a backbone.

If <u>an animal is a vertebrate</u>, then <u>it has a backbone</u>.

✔ *Checkpoint* **Write the conditional statement in if-then form.**

1. All triangles have 3 sides. If a figure is a triangle, then it has 3 sides.	**2.** When $x = 2$, $x^2 = 4$. If $x = 2$, then $x^2 = 4$.

Example 2 *Write four related conditional statements*

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Olympians are athletes." Decide whether each statement is *true* or *false*.

Solution

If-then form <u>If you are an Olympian, then you are an athlete. *True*, Olympians are athletes.</u>

Converse <u>If you are an athlete, then you are an Olympian. *False*, not all athletes are Olympians.</u>

Inverse <u>If you are not an Olympian, then you are not an athlete. *False*, even if you are not an Olympian, you can still be an athlete.</u>

Contrapositive <u>If you are not an athlete, then you are not an Olympian. *True*, a person who is not an athlete cannot be an Olympian.</u>
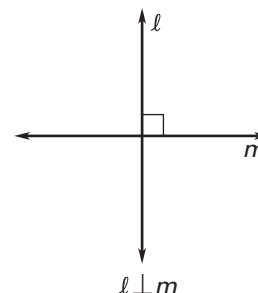
PERPENDICULAR LINES

Definition If two lines intersect to form a __right__ angle, then they are perpendicular lines.

The definition can also be written using the converse: If any two lines are perpendicular lines, then they intersect to form a __right__ angle.

You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.

ℓ
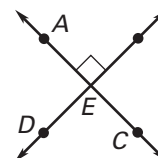
m

$\ell \perp m$

Example 3   *Use definitions*

Decide whether each statement about the diagram is true. *Explain* your answer using the definitions you have learned.

a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

b. $\angle AED$ and $\angle BEC$ are a linear pair.

A

D *E*

C

Solution

a. The statement is __true__. The right angle symbol indicates that the lines intersect to form a __right__ angle. So you can say the lines are __perpendicular__.

b. The statement is __false__. Because $\angle AED$ and $\angle BEC$ are not __adjacent__ angles, $\angle AED$ and $\angle BEC$ are not a __linear pair__.

Example 4   *Write a biconditional*

Write the definition of parallel lines as a biconditional.

Definition: If two lines lie in the same plane and do not intersect, then they are parallel.

Solution

Converse: __If two lines are parallel, then they lie in the same plane and do not intersect.__

Biconditional: __Two lines are parallel if and only if they lie in the same plane and do not intersect.__

✅ *Checkpoint* **Complete the following exercises.**

3. Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Squares are rectangles." Decide whether each statement is *true* or *false*.

If-then form: If a figure is a square, then it is a rectangle. True, squares are rectangles.

Converse: If a figure is a rectangle, then it is a square. False, not all rectangles are squares.

Inverse: If a figure is not a square, then it is not a rectangle. False, even if a figure is not a square, it can still be a rectangle.

Contrapositive: If a figure is not a rectangle, then it is not a square. True, a figure that is not a rectangle cannot be a square.

4. Decide whether each statement about the diagram is true. *Explain* your answer using the definitions you have learned.
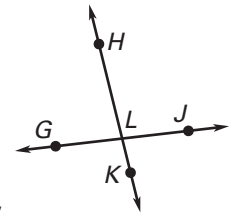
a. $\angle GLK$ and $\angle JLK$ are supplementary.

b. $\overleftrightarrow{GJ} \perp \overleftrightarrow{HK}$

(a) True; linear pairs of angles are supplementary.

(b) False; it is not known that the lines intersect at right angles.

5. Write the statement below as a biconditional.

Statement: If a student is a boy, he will be in group A. If a student is in group A, the student must be a boy.

A student is in group A if and only if the student is a boy.

Homework