2.5 Reason Using Properties from Algebra

Goal • Use algebraic properties in logical arguments.

Your Notes

ALGEBRAIC PROPERTIES OF EQUALITY

Let a, b, and c be real numbers.

Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $ac = bc$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then a can be substituted for b in any equation or expression.

Example 1 ***Write reasons for each step***

Solve $2x + 3 = 9 - x$. Write a reason for each step.

Equation	Explanation	Reason
$2x + 3 = 9 - x$	Write original equation.	Given
$2x + 3 + x = 9 - x + x$	Add x to each side.	Addition Property of Equality
$3x + 3 = 9$	Combine like terms.	Simplify.
$3x = 6$	Subtract 3 from each side.	Subtraction Property of Equality
$x = 2$	Divide each side by 3.	Division Property of Equality

The value of x is 2.

Your Notes

DISTRIBUTIVE PROPERTY

$a(b + c) =$ $ab + ac$, where a, b, and c are real numbers.

Example 2 ***Use the Distributive Property***

Solve $-4(6x + 2) = 64$. Write a reason for each step.

Solution

Equation	Explanation	Reason
$-4(6x + 2) = 64$	Write original equation.	**Given**
$-24x - 8 = 64$	Multiply.	Distributive Property
$-24x = 72$	Add 8 to each side.	Addition **Property of Equality**
$x = -3$	Divide each side by -24.	Division **Property of Equality**

Checkpoint **Complete the following exercises.**

1. Solve $x - 5 = 7 + 2x$. Write a reason for each step.

$x - 5 = 7 + 2x$	Given
$x - 5 - x = 7 + 2x - x$	Subtraction Property of Equality
$-5 = 7 + x$	Simplify.
$-12 = x$	Subtraction Property of Equality

2. Solve $4(5 - x) = -2x$. Write a reason for each step.

$4(5 - x) = -2x$	Given
$20 - 4x = -2x$	Distributive Property
$20 = 2x$	Addition Property of Equality
$10 = x$	Division Property of Equality

Your Notes

Example 3 ***Use properties in the real world***

Speed A motorist travels 5 miles per hour slower than the speed limit s for 3.5 hours. The distance traveled d can be determined by the formula $d = 3.5(s - 5)$. Solve for s.

Equation	Explanation	Reason
$d = 3.5(s - 5)$	Write original equation.	**Given**
$d =$ $3.5s - 17.5$	Multiply.	Distributive Property
$d +$ 17.5 $=$ $3.5s$	Add 17.5 to each side.	Addition **Property of Equality**
$\frac{d + 17.5}{3.5} = s$	Divide each side by 3.5.	Division **Property of Equality**

REFLEXIVE PROPERTY OF EQUALITY

Real Numbers For any real number a, $a = a$.

Segment Length For any segment AB, $AB = AB$.

Angle Measure For any angle A, $m\angle A = m\angle A$.

SYMMETRIC PROPERTY OF EQUALITY

Real Numbers For any real numbers a and b, if $a = b$, then $b = a$.

Segment Length For any segments AB and CD, if $AB = CD$, then $CD = AB$.

Angle Measure For any angles A and B, if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

TRANSITIVE PROPERTY OF EQUALITY

Real Numbers For any real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.

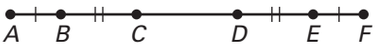
Segment Length For any segments AB, CD, and EF, if $AB = CD$ and $CD = EF$, then $AB = EF$.

Angle Measure For any angles A, B, and C, if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Your Notes

Example 4 ***Use properties of equality***

Show that *CF* = *AD*.



Equation	Reason
AB = *EF*	Given
BC = *DE*	Given
AC = *AB* + *BC*	Segment Addition Postulate
DF = *DE* + *EF*	Segment Addition Postulate
DF = *BC* + *AB*	Substitution Property of Equality
DF = *AC*	Transitive Property of Equality
DF + *CD* = *AC* + *CD*	Addition Property of Equality
CF = *AD*	Substitution Property of Equality

✔ ***Checkpoint*** **Complete the following exercises. In Exercises 4–6, name the property of equality that the statement illustrates.**

3. Suppose the equation in Example 3 is $d = 5(s + 3)$. Solve for s. Write a reason for each step.

$d = 5(s + 3)$	Given
$d = 5s + 15$	Distributive Property
$d - 15 = 5s$	Subtraction Property of Equality
$\frac{d - 15}{5} = s$	Division Property of Equality

4. If *GH* = *JK*, then *JK* = *GH*.

Symmetric Property of Equality for Segment Length

5. If $r = s$, and $s = 44$, then $r = 44$.

Transitive Property of Equality for Real Numbers

6. $m\angle N = m\angle N$

Reflexive Property of Equality for Angle Measure

Homework